

On Re-staging the Universal: Butler, Hegel and Contesting the Closure of Logic¹

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1. Preliminaries

“Formalism”, Judith Butler writes,

is not a method that comes from nowhere and is variously applied to concrete situations or illustrated through specific examples. On the contrary, formalism is itself a product of abstraction, and this abstraction requires its separation from the concrete, one that leaves the trace or remainder of this separation in the very working of abstraction itself. In other words, abstraction cannot remain rigorously abstract without exhibiting something of what it must exclude in order to constitute itself as abstraction.²

The general context of Butler’s essay ‘Re-staging the Universal’ is a consideration of universality in the political realm which, while fully apprised of the ‘false universality’ of colonial and imperial projects,³ wants to restage the universal as a project of cultural translation.

In the essay Butler reads Hegel as calling into question whether formalisms are ever really as formal as they purport to be.⁴ Reading paragraphs 19 to 25 of Hegel’s *Encyclopaedia Logic* she presents Hegel’s approach to universality as proceeding by way of successive revisions of the notion of universality. Thus the form (product) and character of thought are a) universal qua ‘abstract’. But then thinking as activity yields b) the

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² Judith Butler, ‘Re-staging the Universal’ in Judith Butler, Ernesto Laclau and Slavoj Žižek, *Contingency, Hegemony, Universality: Contemporary Dialogues on the Left*, London, Verso, 2000 at 19.

³ Butler, above n.2 at 15.

⁴ Butler, above n.2 at 14–15.

active universal which produces c) a deed as the universal. Thus three different names are offered for a universality, both singular and various, to which is added d) the subject, the pronomial ‘I’ as also the universal. Hegel, she argues, is inhabiting a Kantian voice prior to a critique of Kant for suppressing the internal form of d) — the external form being, here, communality. Taken abstractly, the ‘I’ is pure relation to itself “in which abstraction is made from representation and sensation, from every state as well as from every peculiarity of nature, talent, experience”.⁵ Such a positing of the universal ‘I’ requires the exclusion of what is specific and living from the self and since this too is universal we get a doubling designated in terms of abstract and concrete. In general, Hegel’s point against Kant, according to Butler, is made by showing in various contexts that

when the universal is conceived as a feature of thought, it is by definition separated from the world it seeks to know.⁶

To the extent that freedom of thought guarantees freedom, freedom is defined precisely over and against all exterior influence and this abstract freedom intrinsic to thought, brings a certain hubris, or will to mastery, to be countered by ‘humility’, ‘modesty’ that is attained by immersion in the *matter* itself. “Hegel will conclude”, Butler writes,

that not only is the thinking self fundamentally related to what it seeks to know, but the formal self loses its ‘formalism’ once it is understood that the production and exclusion of the ‘concrete’ is a necessary precondition for the fabrication of the formal. Conversely, the concrete cannot be ‘had’ on its own, and it is equally vain to disavow the act of cognition that delivers the concrete to the human mind as an object of knowledge.⁷

Butler will go on to draw from this her counter to Bataille’s and Derrida’s dubbing of Hegel’s thought as the thinking of mastery (as distinct from sovereignty),⁸ via a consideration of Hegel’s phenomenological (i.e. in *The Phenomenology of Spirit*) linking of universality to reciprocal

⁵ Ibid at 16.

⁶ Ibid at 17.

⁷ Ibid at 18.

⁸ Ibid at 19; see further J. Derrida, ‘From Restricted to General Economy A Hegelianism without Reserve’ in *Writing and Difference*, Alan Bass trans., London, Routledge, 1978, 251–277; Joseph C. Flay, ‘Hegel, Derrida and Bataille’s Laughter’ and Judith Butler ‘Commentary on Joseph Flay’ in *Hegel and his Critics: Philosophy*

recognition and the role of customary practice or *Sittlichkeit* as a substantive rather than formal condition of recognition. From the implicit rejection of transcultural norms in this thinking Butler moves to the performative, cultural translation, as a possible forging of universality which crosses cultures without transcending culture.

2. Dilemma

This paper began its life, some years ago, as an attempt to supplement Butler's essay by distinguishing logical formalisms from the theoretical formalisms of which she is critical. Formal thought, I claimed, does remain rigorously itself, as a practice of mathematical logic, quite simply by adhering to the assumptions, definitions, axioms (if any) and rules it has set for itself. If even so it runs into antinomies and comes up with paradoxical results these should be seen to inhere in something other than the contamination of the abstract by the concrete in the formalisms of which Butler is critical. That something other is the role of contradictions in concept formation. What I was aiming at, on the basis of my own reading of Hegel's *Logic*, was the possibility of turning his idea of a dialectic of pure reason to the task of questioning all forms of authority, including that which may be thought to inhere in classical logic itself.

That remains my aim, but it now seems to me that a distinction between 'logical' and 'theoretical' formalisms made in terms of formal thought 'remaining rigorously itself' is inapposite. The 'I' of a Kantian consciousness, as it clings to Kant's transcendental unity of apperception,⁹ is very much in question in Hegel's critique of the critical philosophy. His attempt to replace Kant's foundational notion of the transcendental unity of apperception with his own logical foundation and so overcome Kantian and Fichtean 'I's is the undertaking of the *Logic*. Now this, I think, is

in the Aftermath of Hegel, ed. W. Desmond, Albany, State University of New York Press, 1989, 163–178.

⁹“It is one of the profoundest and truest insights to be found in the *Critique of Pure Reason* that the *unity* which constitutes the nature of the *Notion* is recognised as the *original synthetic* unity of *apperception*.” G.W.F. Hegel, *The Science of Logic* [1816], trans. A.V. Miller, Atlantic Highlands, New Jersey, Humanities Press International Inc., 1969 at 584; *Wissenschaft der Logik*, Zweiter Teil, *Sämtliche Werke* v.5, Jubiläumsausgabe 4th ed., Stuttgart-Bad Cannstatt, Friedrich Frommann Verlag, 1964 at 15.

what Butler neglects, as I shall argue below. But there is a further problem affecting my own argument. Things have moved on very considerably in the field of formal logic since Hegel's day. If it is now proposed that philosophy and more particularly political philosophy should allow that the logic of any such logical foundation should be presentable, in conformity with that development, as a formal system of mathematical logic, a problem of discipline and practice cannot be ignored.

However interdisciplinary the exchange between formal logic, mathematics and philosophy was in the first decades of the last century, the subsequent development of mathematical logic has been as a mathematical discipline. Such relations as have been maintained with philosophy, whether designated 'philosophical logic' or 'the philosophy of logic(s)' have been very largely in the very tradition of formal logic which Hegel rejected. The situation currently inherited is one in which it can be and is argued that no formal system that is not 'complete' (meaning roughly fully formalisable) should be permitted the designation of (formal) 'logic'. 'Logic' in this view is confined to classical propositional logic and first order predicate logic.¹⁰ This position in debate in philosophy of logic concerning the nature of logic is no doubt conservative, but it sets a parameter of that debate which could be taken as the 'precise' meaning of formal thought 'remaining rigorously itself'. This was and is certainly not the meaning of thought 'remaining rigorously itself' that I intended. On the other hand, if the practice of doing mathematical logic, which standardly includes constructing or working within incomplete systems of higher order logic and set theory, is taken as the practice in which formal thought 'remains rigorously itself' it must be allowed that this is a practice of mathematics.

The stumbling point here is not that it is, as such, not philosophy, but the interaction (or lack of it) between mathematical logic and philosophy. Such interaction would take the form of foundational research but as far as I can see disciplinarity has extended into this field too. Foundations of mathematics remains a (small) field within mathematical logic which may be philosophically engaged. Such engagement however tends to be confined to a specialist philosophy of mathematics that is largely within the analytic tradition. Elsewhere philosophy, has tended toward critique

¹⁰ Susan Haack, *Philosophy of Logics*, Cambridge, Cambridge University Press, 1978 at 6, discussing the view of W.C. Kneale.

or deconstruction of metaphysics or to hermeneutics and taken a variety of ‘ethical turns’ on questions of justification. Alain Badiou’s *L’être et l’événement* (1988)¹¹ with its extended interpretation of ZFC set theory¹² as ontology is a notable exception. But this standard set theory is a classical theory which has guarded itself from paradox by a judicious choice of axioms.¹³ As a form of higher order logic it is designed to avoid the contradictions which a dialectical and speculative logic in the spirit of Hegel seeks to accommodate.

‘In the spirit of Hegel’: can this be claimed? Compared to Descartes, to Spinoza, to Kant, Hegel turns philosophy’s back to mathematics most emphatically, heaps contempt on Leibniz’ “immature” idea of a symbolic universal language of thought and declares the German language preeminently suitable to his enterprise! Given his insistence on the inseparability of form and content, and on (his) dialectical method as the universal aspect of the form of the Notion,¹⁴ to revise Hegel on this point can surely be said to reject his philosophy.

I am drawn two ways by such saying. One inclination is to say yes, but that is no obstacle to finding in Hegel’s thought a questioning of authority and of law that I think comes from his idea of thought’s (dialectical and speculative) logical foundation. Call the resulting discourse an interdisciplinary legal and political ‘theory’ rather than ‘philosophy’. The name is irrelevant to an enterprise that will still, as a matter of its method, require attention to Hegel’s texts and only differs from other interdisciplinary theories of law and politics in its reference to mathematical logic. The other is to ask, counterfactually: how would Hegel have responded to the antinomy that sank Frege’s hopes of proving Kant wrong on the nature of arithmetic by reducing it to a formal mathematical logic? He might have been happily surprised to find his idea of dialectic as a necessary function

¹¹ Translated by Oliver Feltham as *Being and Event*, London, Continuum, 2005.

¹² Zermelo-Fraenkel set theory with the axiom of choice

¹³ Badiou explicitly defends his commitment to classical logic (as against intuitionistic logic) in Meditation 24.

¹⁴ I use the term ‘Notion’, following the English translations of Hegel used, to designate ‘concept’ (*der Begriff*), as distinct from ‘idea’ (*die Vorstellung*, often translated as ‘figurate conception’ or ‘pictorial thinking’ but better understood as a vague, imprecise idea) and ‘Idea’ (*die Idee*, a realised Notion or a Notion that is adequate to its content and so, in Hegel’s sense “the objective truth or the truth as such”) (Hegel, above n.9 at 755; 236).

of reason gain support from an unexpected quarter and revised his own judgement on the means and methods appropriate to accomplishing the aim of his logic.

Between these two responses, the motivation that first prompted my engagement with Butler's essay, still presses. I would like to see political thought 'on the left' engaging with a discipline born of an originally interdisciplinary exchange between formal logic, mathematics and philosophy which has played no small part in the development of the machines which are employed to write, publish and circulate their ideas. No doubt, as Hegel somewhere remarks, one does not have to study the digestive system in order to digest one's food. No more can a demand be placed on political theorists to study recursion theory (or the theory of computability) in order to read and write texts, produced with the aid of a word processor, on articulations of power within a social order. Still, where abstraction and formalisms arising from it are topics in a project of restaging the universal, I see disciplinary and practical barriers but no justification for setting the universality of concepts in mathematical logic beyond the horizon of engagements. I would go so far as to suggest a lordly contempt of slaves, tools and machines within this attitude.

Re-staging the universal, as that is called for in left political theory, does not, in my view, rest with the possibility of re-staging through cultural translation, although it needs that too. It does not rest there because, in my view, (formally, mathematically) logically constructed universals and the hegemony of classical logic should fall within the challenges taken up by such theory. No doubt legal and political theory is far removed from the logical realm. That is to say, the concepts deployed in and more or less systematically organising such theory are both multiply mediated and separated by gaps from concepts, perhaps indicated by the same word (for example, 'reasonable', 'necessity', 'freedom'), which are located in the logical realm. But if Hegel's idea of thought's logical foundation is being taken up, then a question of how and the extent to which pure reason's forms and functions are relevant to such theory arises. It is a question that goes, in Hegel's terms, to how reason and the reasonable are conceived.

If it is being taken up: that is *my* enterprise and it lurches straight back into the stumbling point mentioned. It is *not* what is being taken up by Butler and the theorists with whom she is in dialogue, Laclau and

Žižek. Quite to the contrary, Laclau regards Hegel's philosophy as 'pan-logicism' and while Butler questions that, her arguments for the openness of Hegel's dialectic rest on the impossibility of a purely formal discourse. Pursuing the dictate of the subjective and motivating 'should' of the previous paragraph brings me into conflict with her too. Evidently enough, the difficulty stems from exclusion of a notion of 'the formal' that is applicable to contemporary formal, mathematical logic from the ambit of her claims. But this exclusion disables the argument which I wish to make, namely, that Hegel while rejecting the sense of 'the formal' which separates form from content, *does* intend his *Logic* as a logic of pure thought; as 'the formal' of and in his way of thinking or 'the formal' of pure reason.¹⁵ It seems then that the supplement I envisaged, as called for in political theory, cannot do what I wanted it to do, that is, leave Butler's ideas for re-staging the universal in place — the place being cultural theory — while using Hegel's idea of thought's logical foundation to give a critical standpoint *vis à vis* authority claims *without* distinguishing 'logical' and 'theoretical' formalisms.

A case then of damned if one does and damned if one doesn't? Indeed: a dilemma of the times.

3. Hegel: Thematically

Philosophical thinking in general is still concerned with concrete objects — God, nature, spirit: but logic is concerned only and solely with these thoughts *as thoughts*, in their complete abstraction.¹⁶

To my mind, Hegel's thinking has both its radically emancipatory moment and its logical character in this aspiration. His idea for a dialectical and speculative logic is, as I read him, a foundational idea that pushes Kant's

¹⁵ "Logic being the science of the absolute form, this formal [science] (*dies Formelle*), in order to be *true*, must possess in its own self a *content* adequate to its form; (Hegel, above n.9 at 594; 29). Translation of *dies Formelle* as "this formal science" reads well but occludes the substantive of the German text, *das Formelle*, 'the formal'.

¹⁶ G.W.F. Hegel, *The Science of Logic* [1812], trans. A.V. Miller, Atlantic Highlands, New Jersey, Humanities Press International Inc., 1969 at 34; *Wissenschaft der Logik*, Zweiter Teil, *Sämtliche Werke* v.4, Jubiläumsausgabe 4th ed., Stuttgart-Bad Cannstatt, Friedrich Frommann Verlag, 1964 at 24.

transcendental turn to a fully logical turn via arguments on two fronts. On the one hand, it takes issue with Kant for leaving formal logic outside the scope of his first *Critique* and so with the distinction between formal or general and transcendental logic in the critical philosophy. On the other hand, he takes issue with the irresolution of Kant's turn against the idea that thought is dependent for its content, albeit as mediated by the pure forms of intuition (time and space), on sensible objects.

Hegel's difference with Kant as regards the nature of 'logic' is formulated as the difference between regarding logic as a canon of judgement — in Kant's own terms, a priori principles of how the understanding ought to think¹⁷ — and as an organon or tool for the production of objective insights. The argument is about the nature (and so the authority) of reason, that is, thinking in terms of relations between concepts, and it takes in reason's relation to the understanding, that is thinking in terms of bounded concepts. It is a basic and intractable difference.¹⁸ The merely regulative role given to reason in Kant's philosophy is Hegel's abiding objection to it. The power of thinking the unconditioned, which for both is reason's claim, is denied access to 'truth' in its cognitive (theoretical or speculative) exercise in Kant's philosophy. Confined in this power to what may be learned in its pure practical exercise, reason's most valuable accomplishment, the reasonable, turns out to be an 'ought to be' which 'is' not in this world and is thus unknowable by reason in its cognitive moment.

Hegel states his *logos* preliminary to the body of derivations of the *Logic*,¹⁹ as "the objectivity of illusion and the necessity of contradiction that belongs to the nature of thought determinations". It is, he says "nothing else but the inner negativity of the determinations as their self-moving soul, the principle of all natural and spiritual life." Placed into an homage to Kant for freeing dialectic from its reputation as arbitrariness

¹⁷ "Logic is a science of reason not only as to mere form but also as to matter; a science a priori of the necessary laws of thinking, not, however, in respect of particular objects but all objects generatim; it is a science, therefore, of the right use of the understanding and of reason as such, not subjectively, i.e. not according to empirical (psychological) principles of how the understanding thinks, but objectively, i.e. according to a priori principles of how it ought to think." Immanuel Kant, *Logic* [1800], trans. Robert S. Hartman and Wolfgang Schwarz, New York, Dover Publications, 1974 at 18.

¹⁸ See above p.65.

¹⁹ For the distinction made here see below n.32

and showing it to be a “*necessary function of reason*”,²⁰ and formulated as an extension of Kant’s Antinomy of Pure Reason, this statement indicates Hegel’s standpoint. He envisages a science of this *logos* — a logic in a classical philosophical sense — in which contradictions are relevant in a way that is both limitative (negative, dialectical) and constitutive (speculative).

I thus take Hegel at his word as intending his logic *as a logic* that will *replace* all previous metaphysics.²¹ The first part, the Objective Logic with its purported derivation of the categories of the understanding, takes the place of ontology and ontotheology. The second part, the Subjective Logic, has the logical forms (of judgement, syllogism, theoretical and practical reasoning) as its subject matter. That is for Hegel the activity of the Notion which, as ‘derived’ in the final transition of the Objective Logic (so as the logical Notion or concept of concept) is *the form* of the conscious and self-conscious subject. Form and content are dialectically related in Hegel’s thinking. They are not identical and nor are they independent of realm. The forms of consciousness and self-consciousness into which *Geist* or Spirit (‘the Hegelian subject’ or ‘thinking subject’ in Butler’s terminology) enter, are logical forms, products of pure thought. In that sense of ‘subject’ that is always already embodied in the social relations of a place and time, Hegel’s *Logic* is subjectless. In a metaphor that is not without its own interest in contexts of logic and politics, he writes of the Subjective Logic that its “task is to remodel an ancient city, solidly built, and maintained in continuous possession and occupation”.²² He is talking about the Aristotelian tradition of formal logic that he has elsewhere described as a “heap of dead bones” a “dull and spiritless reckoning”: an incarnation *par excellence* of the formalism of which he was critical, but which is not the less part of the heritage.

Hegel certainly sticks to a resolve to make no use of much of *that* formal logic — and it might be added that the tradition returns the compliment by ignoring Hegel — but this is not to say that the logic he intends does not challenge it.²³ There is perhaps disinterest in, rather than a challenge to, the *correctness*, within its own frame, of the formal logic of

²⁰ Hegel, above n.16 at 56; 54.

²¹ Ibid at 63f.; 64f.

²² Hegel, above n.9 at 575; 3.

²³ In some interpretations, Hegel is read as making no challenge to “ordinary logic”; see e.g. Terry Pinkard, ‘A Reply to David Duquette’ in *Essays on Hegel’s Logic*, ed.

his day. Rather, the frame itself, with its separation of form and content, is rejected. Hegel's 'logic' is framed as an encompassing unity within which form (the Notion, universality) and content ('truth' in Hegel's sense) are inseparable *and* for which the method is the universal aspect of the form. In Hegel's own terms the task undertaken in the Subjective Logic is "to go further [than the Aristotelian undertaking] and to ascertain both the systematic connection of these forms and their value".²⁴

Given that Hegel has scant interest in and correspondingly scant specialist knowledge of the tradition of formal logic, the subsequent transformation of that tradition by mathematical logic is somewhat to the side of his thought. What that transformation brought about and continues to bring about, are new questions, questions that are addressed to classical logic. Its so called 'unquestionability', its status in Kant's eyes as a complete and perfect science, has become an anachronism. Further, updating to one of the seminal contributors to that transformation, the universality of (classical) logic as Frege (and Russell and Whitehead) conceived that,²⁵ came at the price of provable contradictions or antinomies when, in pursuit of the logicist attempt to reduce arithmetic to logic, certain axioms governing concepts were included. No doubt the import of the logical and set theoretical antinomies can be and standardly is pushed away and minimised, but that is not to say that this strategy is the best or even a good one. Indeed, explicit or implicit claims that this is the most 'reasonable' strategy, or that it serves 'us' and 'our science' best, are what I am opposing. Possible attitudes to contemporary scientific practices and institutions do not lie on a linear scale between unquestioning acceptance and horrified rejection. Questions of justification, community and objectivity in thought raised by Hegel's idea of thought's dialectical and speculative logical foundation considered from a perspective provided by current research in non-classical logics, open other possibilities. It is difficult to gain and communicate that perspective, but I do not accept the authority of those who declare it impossible or the judgement of those who think it unnecessary. Even as a matter of determining which logic is

G. di Giovanni, New York, State University of New York Press, 1990, 17–25 at 19f. The trouble is that what is meant by "ordinary logic" is entirely unclear.

²⁴ Hegel, above n.9 at 595; 31.

²⁵ Jean van Heijenoort, 'Logic as calculus and logic as language' *Synthese* 17 (1967), 324–330.

best suited for which scientific or technological endeavour the latter field of research works in the breach of the universality of classical logic.

So far as Hegel's idea of foundation and the questions raised by it are concerned, the relationship between the theory of knowledge or justification of *The Phenomenology of Spirit* and the poiesis and practice of the *Logic*, and the particular function that the *Logic* has within Hegel's system of philosophy are basic. In describing the 'result' of *The Phenomenology* as the presupposition of a presuppositionless logic, I take Hegel to be asking his readers to think twice about the relationship in question.²⁶ In my view, Hegel takes *The Phenomenology* to have shown that *its* presupposition or starting assumption, that according to Hyppolite of all theories of knowledge,²⁷ namely the distinction between subject and object (correlatively knowledge and being, in itself and for itself, certainty and knowledge) has been shown to be inadequate to science.²⁸ The standpoint of absolute knowledge absorbs this distinction in a double character of its own. It is a standpoint of a journeying consciousness which has experienced a range of attitudes to its desires and their reversal, and knows itself as the recollecting totality of that experience. It is also the abstract concept of pure science, of 'logic' in Hegel's sense, conceived but not yet realised by a derivation of the determinations of pure thought. The poiesis and practice of the *Logic* is the realisation of that concept *in the logical realm*: a realm that is a construction of thought and which contains no objects other than objects which thought gives to itself, namely concepts and operations on and with those concepts (judgement, syllogism, reasoning). Implicit, to my mind, is the thesis that it is *only* in the logical realm, only where thought by an artifice of idealisation, constructs a realm of absolute freedom, where it is, so to speak, alone with itself, that the ideal unity of theory and practice of knowledge is realisable.

The particular function of this part of the system is to provide thought's logical *foundation*, in a sense of 'foundation' that has its theory of justification in an epistemology and the activity of providing the foundation in logic. In this respect it is a departure from the *kind* of foundation that Kant sought. It presupposes the possibility of objective

²⁶ Hegel, above n.16 at 68; 71.

²⁷ Jean Hyppolite, *Genesis and Structure of Hegel's Phenomenology of Spirit* [1948], Evanston, Northwestern University Press, 1974 at 575.

²⁸ William Maker 'Beginning' in *Essays on Hegel's Logic*, above n.23 at 27–43.

knowledge (science) but not, as Kant supposed, as already instantiated in mathematics and theoretical physics. Rather, at the beginning of the *Logic*, it is a foundation that is yet to be provided, although it admits no doubt that it can be provided in the logical realm.

4. Hegel: Abstraction and Abstraction

There is a sense in which philosophy throughout Hegel's encyclopaedic system is 'logic', (a sense that leads Hao Wang to liken Hegel's conception of 'logic' to that of Wittgenstein in 'On Certainty'²⁹). Philosophy, in Hegel's conception of it is throughout concerned with the Idea albeit in its different modes (*Weisen*) of existence, and throughout concerned with the 'derivation' of the categories for 'objective' knowing of a given realm. The logical mode of the absolute Idea is however its universal mode. Thought is concerned with itself in the *Logic*, with its own forms and functions, not with its modes of existence (*Dasein*) in the world, in the realms of nature and spirit.³⁰

The transitions in Hegel's *Logic* are not and are not intended to be derivations in the sense of inferences drawn on the basis of rules of a formal logic, old or new. They are 'logical' in the sense that they purport to follow a movement of thought thinking thought by a method that takes nothing from outside the realm of pure thought other than the initial intuition of how, practically, that can be done. True enough, as Laclau says in his discussion of panlogicism,³¹ the account of the method of the *Logic* comes only at the end, after the last transition (from the practical or objective Idea or Idea of the good, to the absolute Idea). The body of derivations which are the practice of 'doing logic' in Hegel's sense,³² are however a *use* of the method. Evidently enough, some idea of how to perform the first 'derivation' (of Becoming from Being and Nothing) must precede it. Differently put: Hegel must have had some idea of how that 'complete

²⁹ Hao Wang 'What is Logic' *The Monist* 77 (1994), 261–277.

³⁰ Hegel, above n.9 at 824–5; 328.

³¹ 'Identity and Hegemony' in *Contingency, Hegemony, Universality*, above n.2 at 61.

³² Cf. Hans-Georg Gadamer, 'The Idea of Hegel's Logic' in *Hegel's Dialectic: Five Hermeneutical Studies*, trans. P. Christopher Smith, New Haven and London, Yale University Press, 1976 at 86, for properly hermeneutical comments on what "can properly be called Hegel's text".

abstraction' from the concrete objects of philosophical thinking was to be achieved.

This is the issue addressed in the chapter of the Greater Logic 'With what must science begin?'. Apparently ambiguous, its placement (after the Prefaces and Introductions; before the derivations) is the structural counterpart in that text to the final chapter which gives a reflective account of the method. And for all that I think Hegel judges badly (i.e. on the basis of prejudice and philosophical hubris) and wrongly (in terms of choosing the tool he needs for the end in view) in his lofty dismissal of Leibniz' ideas for using mathematical methods in the pursuit of logic, he is not without his insight here. Taking ordinary language as a given basis, Hegel takes predication in terms of the copula 'to be', and objectification as the turning of a propositional form (predicate or concept) into an object, as operations whereby thought is able to give an object to itself and thus to come up with Being ('is' turned to 'isness') as a meaningless object and beginning of his science.

Whatever richer names be given to [the beginning of science] than is expressed by mere *being*, all that can be considered is *how such an absolute enters into the thinking cognition* [predication V.K.] *and into the expression of this cognition* [objectification V.K.] (my emphasis).³³

I do not claim that Hegel is seeing other than through a glass darkly. I do not deny that this interpretation is made from a perspective provided by contemporary mathematical logic with its use of formalised operations

³³ Above n.16 at 77; 83. Dieter Henrich in 'Formen der Negation in Hegels Logik' in *Seminar: Dialektik in der Philosophie Hegels*, ed. Rolf-Peter Horstmann, Frankfurt am Main, Suhrkamp, 1978, 213–229, draws attention to the '*substantivierte Aussageform*' (propositional form turned noun) as a basic operation in Hegel's logic, but he takes it as converting 'not' to 'nothing'. Yet Being is Hegel's first category and it is, so Hegel, pure thought's capacity to give itself an object that distinguishes philosophy from the other sciences (G.W.F. Hegel, *Logic: Part One of the Encyclopaedia of the Philosophical Sciences* (1830), trans. W. Wallace, Oxford, Clarendon Press, 1975, §17; *Sämtliche Werke* v.8, Jubiläumsausgabe 4th ed., Stuttgart-Bad Cannstatt, Friedrich Frommann Verlag, 1964. He also (in a *Zusatz*) speaks of making 'is' an object of investigation (Ibid §24Z at 40; 88): "... Being is a pure thought-determination: yet it never occurs to us to make 'is' (*das Ist*) an object of our investigation."

of predication and abstraction (objectification).³⁴ I do place quite some weight on this beginning. Structurally, and in terms of clear acknowledgement of *The Phenomenology of Spirit* as the presupposition of a presuppositionless logic,³⁵ this chapter sees Hegel *constructing* a realm of pure thought; finding the means whereby thought can make itself its own object; can turn itself back on itself. It is a capitalisation on the increased degree of reflexivity in Kantian philosophy that turns reflexivity to self-reference with the aim of establishing the universality of thought in its most extensive freedom. Seen again from the perspective of contemporary logic the antinomies and undecidability results of logical self-reference may be seen as vaguely and imprecisely anticipated.³⁶

I am following through here on that aspect of the standpoint of absolute knowing that is Hegel's idea of and for his *Logic*. Objective knowing in Hegel's thought depends neither on the distinction between content and form characteristic of traditional formal logic nor on a process of abstraction appended to the ontological view of empiricist realism (the view that the material given by intuition and representation is real in contrast to the Notion) and its correlative characterisation of the empirical (as 'concrete') over the ideal (as 'abstract'). In Hegel's eyes,

[i]n this view, [the view of empirical realism V.K.] to abstract means to select from the concrete object for *our subjective purposes this or that mark* without thereby detracting from the worth and status of the many other properties and features left out of account; on the contrary, these as *real* retain their validity completely unimpaired, only they are left yonder, on the other side; thus it is only the *inability* of the understanding to assimilate such wealth that compels it to content itself with the impoverished abstraction.³⁷

³⁴ This interpretation of Hegel's beginning has been much discussed with Uwe Petersen whose non-classical symbolic logic 'in the spirit of Hegel' and whose explanations and guidance concerning that and other aspects of contemporary mathematical logic have enabled me to gain that perspective. For a presentation of that logic: Uwe Petersen, 'Logic Without Contraction as Based on Inclusion and Unrestricted Abstraction' *Studia Logica* 64, (2000), 365–403. For my own perspective see the Appendix to this paper, below.

³⁵ Ibid at 68; 71.

³⁶ See J.N. Findlay, above p.78.

³⁷ Hegel, above n.9 at 587; 20.

Hegel opposes this view with the idea of abstraction as a grasping of what is essential in the ‘matter’ with which thought is engaged be that, in the logical realm, itself, or in the realms of nature and spirit, the sensuous and sensuous-supersensuous material of appearance.

As regards Butler’s arguments: the ‘self’ of that ‘itself’ (what she terms ‘the formal self’) in the *Logic*, and so as a ‘derived’ and in that sense justified concept, is the Notion in its logical form. It is neither reducible to method nor capable of being thought without a method: content, form and method are inseparable; their separation *is* ‘formalism’. In so far as that method involves objectification (‘abstraction’ in the context of mathematical logic³⁸) it is not abstraction *from* anything. It is constructive rather than reductive in character. Butler is quite right in moving back to *The Phenomenology* to link universality (as variously manifested in the logical Notion) to reciprocal recognition and the role of customary practice in presenting ‘the thinking subject’ (the Notion that thinks as situated, embodied, sociable human individuals). Hegel indicates this within his logical dialectic by incorporating into the *Logic* a section of *The Phenomenology* portraying the unresolved contradiction of self-consciousness.³⁹ What I think she neglects with her claim that formal thought cannot remain rigorously itself without displaying the contamination of the excluded concrete, is the extension of Kantian reflexivity to self-reference within a purely ‘logical’ realm. A ‘doubling effect’, paradoxes and antinomies that may attend on self-reference and are effects of self-reference rather than the contamination of an excluded ‘concrete’, is thus left out of her account of Hegel’s logical thought.

In conceptual terms the loss is of the notion of ‘formal’ of which Hegel writes with reference to ‘the formal’ (*das Formelle*) of *his* logic.⁴⁰ It is a different conception of ‘formal’, or perhaps one should say it is a different way of thinking ‘formal’, from that which inhabits the tradition of formal logic. Now contemporary mathematical logic is no doubt a formulation of formal logic.⁴¹ In that dimension, it retains and continues the tradition

³⁸ See Appendix, point 5, below.

³⁹ Ibid at 820; 323. It is the one point, within the *Logic*, at which phenomenological and logical dialectics touch.

⁴⁰ Ibid at 594; 29. And see above n.15.

⁴¹ “Mathematical logic, which is nothing else but a precise and complete formulation of formal logic, has two quite different aspects. On the one hand, it is a section of mathematics treating of classes, relations, combinations of symbols etc instead of

and the notion of ‘formal’ embedded or inscribed in it. But it also emerged in and as a radical transformation of the tradition. What has been brought to the old formal logic is a method of formalisation which, in drawing on mathematical methods and reasoning, has encased ‘the formal’ of formal logic in a new, mathematical discipline.⁴² Formal logic has been taken over by mathematics. I do not claim that Hegel’s concept of ‘formal’ has won out, not the least because Hegel’s thought contributed nothing to the transformation: it took place, as I said, somewhat to the side of that thought. But the transformation has seen both unanticipated results (antinomies), and new concepts (completeness, incompleteness, consistency, undecidability) which, applying to formal systems themselves, indicate a content of greater complexity than previously envisaged. Any formal mathematical logic ‘in the spirit of Hegel’ would, in order to be a formal mathematical logic, have to be constructed and presented in accordance with the requirements of that discipline and be open to the charge that it is not ‘in the spirit of Hegel’. Leaving that debate to the proprietary minded, my point here is that the burgeoning complexity of formal mathematical logic is not inconsistent with Hegel’s claim that

this [his] notion of formal (*dieses Formelle*) must be regarded as possessing richer determinations and a richer content and as being infinitely more potent in its influence on the concrete than is usually supposed.⁴³

The conceptual loss that I am asserting here deprives theory of a question that can and in my view should be put to understandings of Hegel as ‘panlogistic’, as regards its ‘logistic’ component. What conception of ‘logic’ and what concrete logical system for doing logic is being assumed?⁴⁴ Second, and as regards the ‘pan’, I think Butler is constrained by her arguments to pass over Hegel’s assertion of the *independence* of the logical Notion from its modes of existence in nature and Spirit. Yet this is essential to the alternative conception of ‘formal’ that is in question. It

numbers, functions, geometric figures etc. On the other hand, it is a science prior to all others, which contains the ideas and principles underlying all sciences” (K. Gödel, ‘Russell’s mathematical logic’ (1944) in Kurt Gödel, *Collected Works*, vol.II, ed. S. Feferman *et al*, Oxford, Oxford University Press, 1990, 119–141 at 119).

⁴² See Appendix, point 1, below.

⁴³ Hegel, above n.9 at 594; 29 [modified translation V.K.].

⁴⁴ Cf. Thomas Sören Hoffman, *Georg Wilhelm Friedrich Hegel: eine Propädeutik*, Wiesbaden, Marixverlag, 2004 at 381.

is an independence, *as constituted*, and *not* by the reductive abstraction involved in grasping what is essential to a matter in hand: not then by the concept of abstraction which Hegel offers in the place of the criticised Kantian notion, “the sublation and reduction of sensuous material to its essence”.⁴⁵ There is no such material to be dealt with in logic. The constitutive abstraction is the objectification. It is thus, it seems to me, that the idea that Hegel wishes to comprehend everything about everything that exists — stones, states, situations, contingencies, discourses, emotions, attitudes, everything — in terms of the logical Idea is countered.

Still, there is something else to the charge of ‘panlogicism’ in this exchange: something that the term, taken less literally or analytically, marks. I think it to involve the charge of claiming to have deduced the One True System and I think that Hegel is in principle committed to this claim.⁴⁶ Further in taking Hegel’s idea of a dialectical and speculative logical foundation of thought seriously I am associating myself with such a commitment. I also think it not inconsistent, indeed dependent on, an idea of ‘logic’ as an open dialectic of concept formation. In a world characterised by changeability, whatever domains of static truths may be established within it, a logic that would provide thought with the kind of foundation Hegel envisages cannot seal itself into a completed system. If

⁴⁵ “Abstract thinking, . . . is not to be regarded (*zu betrachten*) as a mere setting aside of the sensuous material, the reality of which is not thereby impaired; rather is it the sublation and reduction (*das Aufheben und die Reduktion*) of that material as mere *appearance* to the *essential*, which is manifested only in the *Notion*” (Hegel, above n.9 at 588; 20–1). It might be objected that Hegel’s selection of predication is reductive, but the place of natural language in Hegel’s philosophy is not that of ‘the material’ to which he is referring here.

⁴⁶ The original charge of ‘panlogicism’, a term coined in the early twentieth century by Hermann Glockner, (as distinct from ‘panlogism’ which is an earlier and different charge) attributes to Hegel a form of idealism that is committed to a disembodied ‘mind’ as the substance or substratum or essence of being (*The Cambridge Dictionary of Philosophy*, Cambridge, Cambridge University Press, 1995 at 315). As charged by Laclau, Hegel’s ‘panlogicism’ is his “project of presuppositionless philosophy” (‘Constructing Universality’ in *Contingency, Hegemony, Universality*, above n.2 at 306, n.2). From his fuller discussion (in the same volume, ‘Identity and Hegemony’ at 59f.) the “closed totality” of the absolute Idea beyond which “no further advance is possible”, in combination with the necessary rather than contingent nature of Hegelian transitions, seems to constitute the gravamen of the charge. I think with Butler that the meaning of the term is very unclear. My formulation attempts to get at what actually is being objected to here.

it did, it would become redundant or lose its character as an organon for the production of objective insights and become a canon of how we ought to think: something like a catechism or diamat.

In principle; and in practice? First a further clarification of the principle; I am contemplating a conception of ‘logic’ as an open dialectic of concept formation: a dialectic within which the totalities (‘universals’) that concepts are and the totalisations over a finite or indeed infinite class of particulars (including concepts) that yield further ‘higher order’ concepts (such as the universal equivalent in Marx’s general form of value⁴⁷ or transfinite numbers in Cantor’s set theory⁴⁸) extend, inexhaustibly, the logical realm. But now back to practice; does not Hegel close the logical realm, exhaust it in or in the name of the absolute Idea?

Yes or no: this staple of those for whom Hegel is panlogistic is like the repetition of Hegelian dialectic as a triadic movement of thesis, antithesis and synthesis.⁴⁹ One can put much scholarly exegesis against them, rustle up this reading and that reading, get into the endless, useless exchange of charge and complaint — misreading, misinterpretation, misunderstanding — that is the bubble-gum of academic discourse. They do not go away and I think they have less to do with Hegel than with questions of the commitments of those who are convinced that this is the truth about Hegel. I do not mean this dismissively. One can jump onto band wagons and perform well in current debates by following the fashions of philosophical and social theoretical discourses, but one cannot think freely and originally without commitments of a kind that make up the normativity of theory. Such commitments need not be dogmatic. They may be relativised to take account of competing commitments and to permit pragmatic moments within a theoretical praxis of justification and critique. Suspension of commitment for the purpose of following a theory or argument moving off from different commitments is a possible and valuable technique of theoretical engagement. A kind of translation between framings might

⁴⁷ Karl Marx, *Capital* I, trans. B. Fowkes, London, Penguin, 1976 at 157.

⁴⁸ An accessible account is given in Martin Davis, *The Universal Computer: the Road from Leibniz to Turing*, New York, W.W. Norton & Co., 2000 at 69–73.

⁴⁹ See e.g. I. Grattan-Guinness, *The Search for Mathematical Roots: Logic, Set Theories and the Foundations of Mathematics from Cantor through Russell to Gödel*, Princeton, Princeton University Press, 2000 at 72; and in a quite different context Anne Bottomley, ‘Shock to thought: an encounter (of a third kind) with legal feminism’ *Feminist Legal Studies* 2004, 12: 1–37.

reveal that differences go more to expression than substance. But all this said commitments are part and parcel of serious theoretical work.

Hegel is committed to the independence of the logical realm from the realms of nature and social and cultural life (spirit).⁵⁰ It is a commitment to the possibility of constituting that realm by “complete abstraction”: by an artifice of idealisation which I have little difficulty in crediting, albeit as an in-principle idea which would need mathematical techniques to be doable in practice. I have my own difficulties with the absolute Idea and they come back, as far as I can see, to the point that it is effectively a denial of that need. Hegel’s conception of philosophy, as regards content and end, is one that classes it with art and religion as distinct from the particular sciences. But it has an elevated status within this class on account of its conceptuality.

Philosophy has the same content and the same end as art and religion; but it is the highest mode of apprehending the absolute Idea, because its mode is the highest mode, the Notion. Hence it embraces those shapes of real and ideal finitude as well as of infinitude and holiness, and comprehends them and itself.⁵¹

I find it hard to say what actually the stakes are here. Value? Power? Authority? Degree of abstraction? All of these? More too: the genius of those individuals who can think thus abstractly and of those peoples whose cultural possession it is? But if this is Hegel’s logocentrism and if it is inseparable from the ethnocentrism of the historical narrative that forms part of his thinking, I don’t find logophobic solutions any advance in this predicament.

Nor, on my reading, does Butler. Rhetoric rather than logic is her tool, but just in re-staging the universal she is engaging the various conceptualisations of the universal in Hegel, staying short perhaps of the absolute Idea. That is not where our difference lies for holiness (*Heiligkeit*) as attributed to the absolute Idea is problematic for me too. The elevation of the divine above the mundane is a figure which Hegel reiterates from

⁵⁰ Hegel, above n.9 at 586; 18: “[T]he Notion that is self-conscious and thinks pertains solely to spirit. But the logical form of the Notion is independent of its non-spiritual, and also its spiritual shapes. The necessary premonition on this point has already been given in the Introduction. It is a point that must not wait to be established within *logic* itself, but must be cleared up *before* that science is begun.”

⁵¹ *Ibid* at 824; 328.

beginning to end of his *Logic*. I think it compromises the emancipatory potential of his logical and metaphysical ideas and I think it implicated in that lordly contempt for slaves, tools and machines that thinks in terms of higher and lower forms of life and disparages the merely calculative. The task, as I see it, is to get away from this figure of elevation without, in a fit of pathos, dumping the *logos* into the dustbin of cultural prejudice and conceit. Again, however it is not such pathos in which I see Butler as indulging. My point as regards her, is rather that questioning the science of the *logos*, logic, must meet it where it is: in abstract, formal thinking, or rather, to keep hold of differing notions of ‘formal’ that I am seeking to elucidate, thinkings.

Philosophy, in the tradition inaugurated by Kant — that is, philosophy that holds to the idea of metaphysics as the urge to think the unconditioned (or absolute, or infinite, or totality) — adheres, not to all, but certainly to some of Hegel’s ideas if it admits the experience of spirit in the development of mathematics and mathematical logic gained after Hegel’s times.

5. Questioning authority

I am thinking of authority in general in terms of effective power of determination which lays claim to being justified. If it manages to bring that claim within its power, it gains a self-justifying, self-reproducing quality which strengthens it to a degree such that, at least as regards political authority, the need for a standpoint from which challenges to a constituted authority’s claim to justification are made is widely acknowledged. What of the authority of ‘logic’? I have reservations regarding the attribution of authority to logic. It seems to me that they strengthen a common and rhetorical use of claims to the logical necessity of an argument which neglect the relativity of any such necessity to a particular system of logic. I would prefer to say that logic neither has nor needs authority, being merely the consistent application (or construction) of a system of definitions, axioms and rules of inference which have been voluntarily adopted for some purpose or another. Often, I suspect, the appeal invokes, consciously or unconsciously, a claim to the universality of classical logic which I think is outmoded.

Still, the phrase is used by Georg von Wright in asking what gives to deontic logic the authority of Logic?⁵² While I wonder whether, with his capitalised and I assume, totalised ‘Logic’, his question is quite straight faced, he explains that what is being asked after here is a *rationale* for specific principles of a deontic logic. In his view the question demands an answer. To give an answer, he asserts the necessity to step beyond “deontic logic itself” into discursive considerations of reasonableness relating norms to ends and a particular system of deontic logic to the Standard System.

The specific issue that von Wright is dealing with — controversy as to whether a reduction of deontic logic to alethic logic commits the ‘naturalistic fallacy’ — makes it clear that the question of the ‘authority of Logic’ is set into dispute in philosophy regarding the relation between theoretical and practical reason or, in the empiricist tradition, the ‘is-ought’ controversy. The ground then is that of Hegel’s most fundamental disagreement with Kant. Already in the preliminary statement of his *logos* with its homage to and critique of Kant, Hegel’s point of dissatisfaction is formulated. Kant’s dialectical principle is too limited.

[I]f no advance is made beyond the abstract negative principle of dialectic, the result is only the familiar one that reason is incapable of knowing the infinite; a strange result for — since the infinite is the Reasonable — it asserts that reason is incapable of knowing the Reasonable.⁵³

Whether one speaks then of the ‘authority of Logic’ or, as does Hegel, of the ‘authority of reason’ may not make much difference. The ground takes in the politics of theory and, as it mainly concerns me, the politics of a critical (*qua* emancipatory) theory that is still struggling with the Greek heritage of post-Socratic philosophy and Aristotelian logic. On this ground, I am taking the standpoint of Hegel’s ‘absolute’ beginning as regards ‘the formal’ of logic. That is to affirm the gap between logic/ontology and politics, to resist its dissolution (or ‘healing’) in ethical

⁵² Georg von Wright ‘Problems and Prospects of Deontic Logic’ in *Modern Logic — a Survey*, ed. E. Agazzi, Dordrecht, D. Reidel, 1980, 399–423 at 408.

⁵³ Hegel, above n.16 at 56; 54. As von Wright elsewhere points out, one is looking here at different notions of practical inference. Though I would modify his formulation, he deftly grasps Hegel’s thinking of practical inference as a (neglected) departure from both Aristotelian and Kantian paradigms (‘On so-called practical inference’ in *Practical Reasoning*, ed. J. Raz, Oxford, Oxford University Press, 1978, 46–62 at 47).

turns that take the paths of aesthetics or religion *and also* to resist the reduction of the problem to a hegemonic politics of discourse. The latter is my basic difference from Butler and Laclau and to it the constructive moment in formal logic is fundamental. It is that moment that establishes the logical Notion as independent of its modes of existence in nature and in social and cultural life and that brings practice — here practices of formal logical deduction and inference — into my notion of justification and thus ‘foundation’.

Once a plurality of different systems of logic and of ways and frameworks of reasoning is acknowledged, the possibility of challenge to the justificatory claims of or associated with a particular system of logic is opened. It is a space that is especially fragile and vulnerable to the normative closures of pluralism and pragmatism. Let the standpoint from which the claims of a determining power to being justified are challenged be called that of ‘the reasonable’. It could be named after freedom or truth or justice or right, but ‘the reasonable’ has the virtue of bringing things to a point. The idea I find in Hegel is that a self-justifying reason is also a self-critical reason and not quite in the same measure. It is weighted toward the latter as a reason that has learned the vanity of seeking to remove itself and its concepts from the play of reversals and surprises of the kind that Hegel traces in *The Phenomenology of Spirit*. These vain attempts to remove the being-at-odds-with-itself of reason from the reasonable are what Hegel calls ‘formalism’ and against which he proposes ‘the formal’ of his logic with its ‘absolute’ character. The plurality of notions of ‘the universal’ is now *within* Hegel’s *Logic*. This is what Butler takes up but without the further sense of the ‘absolute’ of this formal logic having no ‘outside’, no ‘beyond’. It is a logic of concept formation within a dynamic process: the ongoing activity of the thinking of finite, situated, embodied beings who seek to grasp that process in its excess, by thinking the infinite. My point, to echo Hegel, is that it is least of all, ‘the formal’, the formal notion of the reasonable, reason’s self-conceptualisation, its self-objectification or extension, that should be excluded from a project of re-staging the universal by too limited a view of abstraction. The question here, to my mind, is practical-theoretical or technical. What is the method and means through which this self-justifying/self-critical reason is brought to recognition of its own points of impasse? This to my mind is a question of logic, of which logic, and of which approach to logic.

Let me recapitulate and then take further, the argument begun in the previous two sections. In following Hegel in his ‘logical’ turn of Kant’s transcendental philosophy, but then looking to mathematical logic for the method and means of implementing that turn, it seems that conflicting notions of ‘formal’ are in play: the ‘formal’ that Hegel opposes, that which in the tradition of formal logic separates content and form, and the ‘formal’ which Hegel embraces with his idea of a science of logic. Because mathematical logic does continue the tradition of formal logic albeit shifting its discipline and practice from philosophy to mathematics, it is commonly thought that ‘looking to’ mathematical logic, so far as Hegel’s logical ideas are concerned, is looking in the wrong direction. But this shift in discipline and practice is critical.

Hao Wang comments that “concern with forms, the formal and formalisation is central to the enterprise of mathematical logic”. Discussing Gödel’s incompleteness theorems, he goes on to say that they brought out the central importance of “the interplay of the formal and the intuitive even though the area is devoted to the study of the formal”.⁵⁴ The point of this focus on the formal may just be, as Wang says, “to make precise the concept of formal and thereby be able to reason mathematically about formal systems”. Mathematical and mathematical logical practice is pragmatic. Whichever system enables the pursuit of a particular endeavour will be used. The delicate and foundational question here is what lies between a practice which recognises the moments of indetermination in formal reason and ideological or normative standpoints of pragmatism and pluralism? I do not refuse the benefit of hindsight in approaching this question. For it is, and I use here a pragmatic criterion of outcome, the outcome of this way of thinking, namely that it has added “a new dimension to mathematics”,⁵⁵ which makes me think that it is, despite Hegel’s rejection of mathematical methods, not in principle alien to his conception of logic. It has something of the quality of a method “for the production of objective insights”; a method whereby potentialities immanent in thought can be brought out and articulated by thought making

⁵⁴ Hao Wang, *Reflections on Kurt Gödel*, Cambridge, Mass., The MIT Press, 1987 at 267.

⁵⁵ Hao Wang, *Popular Lectures on Mathematical Logic*, New York, Dover Publications, 1993 at 16.

itself its own object: by thought turning itself back on itself by means of itself.

I say ‘something of’ because Gödel’s result as, so far as I can see, most of the work done in mathematical logic, is a result of and in classical logic. It seems to be on account of his technical virtuosity as a classical, formal, mathematical logician that Gödel managed to skirt the difficulties which classical logic has with handling self-reference and deliver his surprise. My point here is just that this row of adjectives — classical, formal, mathematical — suggests a quite particular logical system. What changes when ‘classical’ is dropped? I am saying that logic’s disciplinary shift and the method of formalisation that accompanied it, brings a notion of ‘formal’ that is not wholly continuous with that which Hegel was opposing.

As a method, formalisation in logic and mathematics underwent a development which is inseparable from the development of mathematical logic and from developments in mathematics in the nineteenth century. The latter, in the later part of the century, saw a reconceptualisation of the very nature of mathematics itself. From being conceived as the science of quantity, the application of mathematical abstractions to the study of mathematical objects began to invest the abstractions with “a life of their own”.⁵⁶ By 1847, George Boole was proposing a definition of mathematics as a general science of symbolic calculi (semiotics). Along a different path, Cantor in 1883 characterised mathematics as

in its development entirely free . . . and only bound in the self-evident respect that its concepts must both be consistent with each other and also stand in exact relationships ordered by definitions, to those concepts which have been previously introduced and are already at hand and established.⁵⁷

The theme of abstractions acquiring a life of their own is also emphasised by Rózsa Péter (sometimes called ‘the mother of recursive function theory’):

Man created the natural number system for his own purposes

But once created, he has no further power over it. The natural number series exists; it has acquired an independent existence.

⁵⁶ Jeremy Avigad and Erich H. Reck, “Clarifying the nature of the infinite”: the development of metamathematics and proof theory’, Carnegie Mellon Technical Report CMU-PHIL-120, 2001, 1–53 at 8.

⁵⁷ Quoted from Avigad and Reck, *ibid* at 9.

No more alterations can be made; it has its own laws and its own peculiar properties, properties such as man never even dreamed of when he created it. The sorcerer's apprentice stands in utter amazement before the spirits he has raised. The mathematician 'creates a new world out of nothing' and then this world gets hold of him with its mysterious, unexpected regularities.⁵⁸

My point here is not just to support my suggestion of discontinuity with conceptions of the formal in the context of the old formal, classical logic. It is also to attempt to lay hold of and bring together two further thoughts. First, that Hegel's *logos* "the objectivity of illusion and the necessity of contradiction inhering in thought determinations" is, as a principle of intelligibility, one that certainly points to the demiurgos-like character of the agency of pure reason, but not one which, as Marx thought, posits the Idea as a demiurgos outside the realm constructed by thought thinking itself. I take Hegel, in his *Logic*, to be working with and within the human capacity for abstraction and idealisation in conceptual thought. The actual or completed infinite, as conceived and used by Cantor in creating his set theory, as distinct from the traditional and Aristotelian potential infinite, is illustrative of the capacity for abstraction and idealisation to which I refer.⁵⁹ That it first gave rise to a theory which became the fundamental discipline for the whole of mathematics; a "completely solid and sound" basis of mathematics as Poincaré at one time declared,⁶⁰ and then, as itself contradictory and in leading Russell to the discovery of his famous antinomy, caused "the whole building to rock",⁶¹ is the (dramatised) phenomenon that leads into the second thought.

As I have said, Hegel's *Logic* is, as regards a thinking subject, subjectless. But the abstract *idea* of his *Logic* is a rethinking of the notion of objectivity emerging from *The Phenomenology*. This idea goes hand in hand with a thinking subject that knows itself as implicated in, not opposed to the 'objects' of its knowledge. It is the idea of a subject constituted in the

⁵⁸ Rózsa Péter, *Playing with Infinity: Mathematical explorations and excursions* [1957], trans. Dr. Z.P. Dienes, New York, Dover Publications, 1976 at 22.

⁵⁹ The other example, used earlier, of a higher order concept, Marx's general form of value, is different in that it is a concept which comprehends an actual social development of practices of production and exchange.

⁶⁰ Abraham A. Fraenkel, Yehoshua Bar-Hillel, Azriel Levy, *Foundations of Set Theory* second rev. ed., Amsterdam, North-Holland Publishing Co., 1973 at 14.

⁶¹ Péter, above n.58 at 229.

experience of a range of attitudes to its desires and their reversal. This thinking subject is, so to speak, paused or suspended, once the logical realm is entered. Its place is taken by method. But the experience of this subject in time is not ‘cancelled’ and that its experience, in the mathematical realm, is a further chapter in its phenomenology is my second thought. Thinking the infinite and its contradictions, and the desire for a system of complete and certain knowledge and its reversal, in the period from Cantor through to Gödel, play out as variations on Hegel’s theme.⁶²

As a layered narrative of the experiences of a journeying consciousness, *The Phenomenology* is a philosophical genre that resiles from an oppositional relation between narrative and conceptual discourse. The *Logic*, certainly, is inserted into the tradition of purely conceptual, that is *formal*, discourse, in which methods of abstraction take the place of narrative form.⁶³ That makes for a gap between the two works and it is, I think, for this reason that the reader is asked to think twice about the relation between them by Hegel’s designation of *The Phenomenology* as the presupposition of a presuppositionless logic.

This thinking twice, this thinking between the phenomenological and the logical is the ongoing demand on thought which is my answer to the question posed above regarding pragmatism and pluralism. It should not be plastered over by words which offer verbal formulae in its place, whether these are Hegel’s words (‘speculative proposition’) or words with current appeal (‘aporetic’, ‘paradoxical’). Butler rests her discussion of Hegel on universality with just such a formula.⁶⁴ Gillian Rose, another inspiring reader of Hegel, does likewise with the repeated formulation of the absolute as that which must and cannot be thought.⁶⁵ It is, to my

⁶² Like the failure of Frege’s project, Gödel’s incompleteness theorems have curious relations with tradition and change. As the use of mathematical methods to construct and investigate formal mathematical systems added a new dimension to *mathematics* and increased mathematical knowledge of formal systems, it radically undermined the idea of foundations and the means of securing foundations for classical mathematics that David Hilbert, at a certain end-point of Kantian ideas of foundation, proposed. See further, the Appendix to this paper below.

⁶³ Hegel, above n.9 at 588; 21.

⁶⁴ “The propositional sense of the copula must be replaced with the speculative one” (Butler, above n.2 at 24).

⁶⁵ Gillian Rose, *Hegel contra Sociology*, London, Athlone, 1981 at 42, 92, 204. Encapsulated in this repeated and developed assertion, is the view that Hegel’s philosophy has no social import if the absolute cannot be thought, followed by the observation

mind, the failure to include that experience of spirit which came with the shift in the discipline and practice of logic from philosophy to mathematics and the accompanying loss or innocence of the tools of formal logic that constrains them.

So how does the relation in question look if it *is* thought twice about, now, almost two hundred years later? It is the relation between a theory of knowledge or justification and a logic. Whether and how Hegel would have taken up and thought that reversal of Leibniz' dream which showed up with the appearance of antinomies and incompleteness in (some) formal systems is and must remain a question. But to my mind, the recollecting totality that Hegel's journeying consciousness knows itself to be, can hardly be thought to have learnt or to remember anything at all if it thinks its ground will not again be pulled from under its feet.

That ground is Hegel's judgement regarding the means and method for attaining the 'complete abstraction' at which he aimed in his logic. The means is natural language. Philosophy, Hegel asserts, has the "right to select from the language of common life which is made for the world of pictorial thinking, such expressions as *seem to approximate* to the determinations of the Notion".⁶⁶ The method must be taken from philosophical not mathematical reasoning. Laclau finds, on the first point, a definitive argument against Hegel's logic being a logic at all.⁶⁷ It is not to my point to argue otherwise if by 'Hegel's logic' is meant the concrete product of Hegel's labours. It is Hegel's *idea* of and for a speculative and dialectical logic that will take the place of metaphysics that I am holding on to. I reject his judgement as overtaken as regards that very purpose. For, what emerges in mathematical logic, through formalisation, is a peculiar content, peculiar limits to the 'precisely defined' notion of the formal itself, which tear at the very way in which 'logic' is thought: tears at its own classical paradigm. This is a phenomenon that so far from refuting Hegel's idea renews the possibility of its realisation.

that the absolute cannot be thought because the dichotomies of concept and intuition, theoretical and practical reason are not transcended. I owe a great deal to Rose's interpretation of Hegel, but I think that the juxtaposition of something that *must* be done (if Hegel's thought is to be relevant and useful) and its present impossibility too short. It sticks in the mystery of the speculative proposition which tries to say and unsay freedom's necessity.

⁶⁶ Hegel, above n.9 at 708; 177.

⁶⁷ Above n.2 at 63.

I am following through on Hegel's thinking to a point where, to my mind, there is a parting of the ways between 'philosophy' and 'logic' as the latter is now constituted and practiced. I do not see how philosophy can be denied the 'right' which Hegel takes it to have without losing its 'self'.⁶⁸ And who knows what Hegel's greater commitment was to: philosophy, his own *Dasein* as philosopher and the sanctity of his judgement regarding method and means, or his idea of a new, non-classical, dialectical and speculative, science of logic? If the latter, he might have blushed at a certain lack of courage of conviction in the power of dialectic to assert itself even in so barren a realm as that of numbers; a certain fear that Leibniz was right and that one needs the imprecision and ambiguity of a natural language to show thought at odds with itself.

I am following it through to a point where it is no longer a question of what Hegel thought or intended, but following it through on just that aspect of his idea of thought's logical foundation that is tied to the experience of spirit. The check, that which holds the absolute in Hegel's thinking of it away from the banality of absolutism, is that experience. Should one say then that the reasonable, reason's fetish form, drawn back into this process, is endlessly, infinitely revisable? In time I think so. But the thought here, and this is where 'fetish' is a misnomer, is the idea of a logic which as reasonable must also be actual: a presented system satisfying the demands of what is currently known of the discipline and practice of logic. Here Hegel failed, on his judgement and according to the demand he himself placed on the reasonable.

That is not to say, on the basis of a different judgement as to means and method, that his idea of a dialectical and speculative logic and the notion of thought's logical foundation that thinks between the phenomenological and the logical has failed or must fail or can only sustain itself in failing. It is to affirm the objectivity of illusion and the necessity of contradiction inhering in the determinations of thought as the principle of intelligibility of objectivity and necessity. It is also to argue for setting the formal science of this *logos* against the tendency of constituted and instituted power to bring the justification of its determinative power within the ambit of that power.

⁶⁸ Cf. Jean-Luc Nancy, *Hegel: the restlessness of the negative*, trans. Jason Smith and Stephen Miller, Minneapolis, University of Minnesota Press, 2002, esp. 19–24.

In the Appendix, I attempt to communicate something of what, as an interdisciplinary researcher, I understand of the discipline and practice of formal mathematical logic. Here I make the plea that motivates this paper. It should be clear that what counts as logical necessity is not independent of the idea of logic involved and gains a formal but concrete sense in a system of logic. It is not only Hegel who rests neither with an abstract idea nor with the currently ruling paradigm. What logic is best suited to which scientific or technological endeavour is a question that is not strange to contemporary logical practice. May it not also be asked which logic is best suited to the endeavour of conceiving objective thinking from the premise that objects that are apprehended and comprehended by thought, whatever their genesis and corporeality, are formed by a subjective activity?⁶⁹ And further: is classical logic suited to reasoning which keeps account of its assumptions from the perspective that an assumption, once used in an act of inference might, like a dollar spent, be so to speak, used up and no longer available in the new situation thought finds itself in?⁷⁰ No doubt there are many reasons for the continuing hegemony of classical logic in both its philosophical and mathematical presentations and some may be ‘good reasons’ for what that is worth. It seems to me that since what counts as a ‘good reason’ must fall under the concept of the reasonable inhering in the way of reasoning formalised in a logic, it is worth very little without the kind of scrutiny given to the Idea of the good that differentiates Hegel’s notion of practical inference from that of Aristotle and Kant, and from pragmatist and neo-pragmatist approaches to justification. Mathematics and mathematical logic has found a freedom in its forms of abstraction and idealisation that can mechanise and simulate itself in some degree. This is the present and there are no gods coming to save us from proprietary abuse of this power. Intervening into this situation, taking responsibility for it, resisting the impotence of beautiful souls and the indifference of ‘hard nosed pragmatists’ who are quite content with the current balance and conduct of power in the world, calls

⁶⁹ It should not be assumed that I am dismissing the pertinence of psychoanalysis and psychoanalytic theory to this question. On the contrary, I see that as also emerging through the experience of spirit following Hegel’s times. My focus however is on the notion of objectivity. I would hope that the lacuna of focus coming from the demands of interdisciplinarity as I apprehend them might be met by collaborative work.

⁷⁰ See the Appendix to this paper, point 3, below, for the background of this question.

the humanities ‘on the left’ to take heed of conceptions and practices of formal logic.

As things now stand, so far as conceptions of logic are concerned, I have come upon this:

[f]or us now, in the last quarter of the twentieth century, the nature of logic — that is, of the discipline of formal, deductive logic — is very largely unproblematic: it is a pure science devoted to the investigation and codification of relations of deductive consequence holding between sentences, or perhaps between the thoughts or propositions they express. And in this connection we understand the need to distinguish (proof theoretic) relations of syntactic consequence and (model theoretic) relations of semantic consequence; there is a general consensus as to how issues concerning, say, formal schemata, calculi, interpretation, truth, validity, consistency and completeness are related one to the other, and today we can be clearer than ever before about how, if at all, the subject matter of logic is related to that of the other disciplines like psychology, mathematics, set theory, ontology, epistemology or linguistics.⁷¹

Clarity, precision and mastery of the nature of logic to hand! Hurrah for ‘us’! And this:

In recent years we have witnessed a very strong and fruitful interaction between traditional logic on the one hand and computer science and Artificial intelligence on the other. As a result, there was urgent need for logic to evolve. New systems were developed to cater for the needs of applications. Old concepts were changed and modified and new concepts came into prominence. The community became divided. Many expressed themselves strongly, both for and against, the new ideas. Papers were rejected or accepted on ideological grounds as well as on technical substance.

In this atmosphere it seemed necessary to clarify the basic concepts underlying logic and computation, especially the very notion of a logical system.⁷²

⁷¹ David Bell, *Husserl*, London and New York, Routledge, 1990 at 87.

⁷² Dov M. Gabbay, Preface to *What is a Logical System*, Oxford, Clarendon, 1994 at v.

Disharmony and dissent — ideology no less! — among workers in the field. And this:

[s]ince the time of Hilbert, no new foundational scheme has been proposed. Certainly people know too much to present a naive ontology of mathematics (and perhaps not yet enough to present a really challenging explanation of mathematical activity).⁷³

And:

[i]t has been a long time since philosophy stopped interacting with logic⁷⁴

Which latter comment is what, were it possible, my plea would change.

APPENDIX

Non-expert observations of mathematical logic

The following makes something of the perspective of mathematical logic than I have gained as an interdisciplinary researcher available to the reader. It is bound to be a bit of a mess: a translation of methods and ideas from a discipline that has, from a certain necessity of abstraction, “gone beyond words”, back into words! It is an absurd undertaking in its way. But I want to underline the sense in which this field, if technical, is also mundane: not mysterious, not a “more than human possession”.⁷⁵ I also want to indicate a path taken into this field that is quite different from the standard introductions and undergraduate courses.

1. **Formalism:** In mathematical logic, ‘formalism’ is a term commonly applied to the product of a particular method of formalisation that developed as part of the emergence of the new discipline. In general, the term ‘formalism’ is synonymous with a range of other terms: calculus, formal system, formal theory, formal mathematics. In so far as formalisation is a general method, thus part and parcel of mathematical logic, this sense

⁷³ Jean-Yves Girard, *Proof Theory and Logical Complexity* v.I, Napoli, Bibliopolis, 1987 at 38.

⁷⁴ J.-Y. Girard, ‘On the Meaning of Logical Rules I: Syntax Versus Semantics’ in *Computational Logic*, ed. Ulrich Berger and Helmut Schwichtenberg, Berlin, Springer Verlag, 1999, 215–272 at 216.

⁷⁵ Hegel, above n.16 at 34: citing Aristotle on logic.

of ‘formalism’ is not negatively connoted. The contribution to methods of formalisation made within the Hilbert school in the first decades of the last century, and the proof theory or metamathematics it established — the logic of logic, as it has been called⁷⁶ — has been a focus of my research.

The method of formalisation developed in this school, proceeds, by means of a “completely symbolic language”, through formal axiomatisation (which renders the primitive terms of the theory meaningless), to divest all other words used in deductions of their meaning, so as to yield an exhaustively defined ‘object theory’ (or ‘formalism’) for purposes of study. The formalisation and study of its product is done from within an informal metatheory.⁷⁷ Its point, so far as Hilbert’s program was concerned, was to prove the consistency of the object theory (some part, eventually, Hilbert hoped, all of classical mathematics) by metamathematical means which, in being restricted to wholly uncontentious methods (so making no use of disputed rules such as *tertium non datur*) would be effective to secure the object theory (which might make use of such rules).

Thus proof theory or metamathematics was developed as means to a particular justificatory end, roughly, to place all of mathematics on a neo-Kantian (methodologicist) foundation that would secure its accomplishments (principally set theory) and ‘the right’ to use principles of classical logic in mathematics. The method in Gödel’s hands, helped rather to undermine that aspiration. In one way of looking at it, it could be said that the master’s tools were indeed used to dismantle the master’s house. Without that procedure this proof would not have been possible and with this proof the last grand theory of foundations of mathematics, that is foundations of the kind that guarantee the truth of existing mathematical knowledge, collapsed. The demands on presentation of a formal logical system remain. Thus

A system of symbolic logic must begin with a list of undefined symbols, a list of formal axioms, and a list of rules of inference.⁷⁸

⁷⁶ J.-Y. Girard, above n.73 at 10: but perhaps more helpfully “Proof theory = logic from a syntactic viewpoint” (ibid).

⁷⁷ Drawing on Stephen Cole Kleene, *Mathematical Logic*, New York, Dover Publications, 1967 at 198f.

⁷⁸ A. Church, ‘The Richard Paradox’ *American Mathematical Monthly* 41 (1934), 356–361 at 356.

That is to say the system is communicated by presentation of a “precise statement of the syntax of the formalism”⁷⁹ It will not be presuppositionless, but it may be the endeavour of setting up a formal logical system to both minimise assumptions and make them explicit as axioms, definitions or precisely formulated rules of the system. One could then say, that constructing a logical system has the character of constructing a game.

In foundations of mathematics, as distinct from the broader context of mathematical logic, ‘formalism’ has a different sense. It may be applied, approvingly, disapprovingly or neutrally to a theoretical standpoint within a debate precipitated by the emergence of antinomies and paradoxes at very basic levels of logic and set theory that took place in the early decades of the last century. In this context, ‘formalism’ and ‘formalists (Hilbert school), ‘logicism’ and ‘logicists’ (Frege, Russell and Whitehead) and ‘intuitionism’ and ‘intuitionists’ (Brouwer and Heyting) entered the lists of the kind of normative or ideological debate familiar in other theoretical endeavours.⁸⁰ The heat seems to have gone out of these debates from about the 1930’s, although foundations of mathematics remains as an area of research. My impression is that the field expanded so considerably from the 1930’s as to overtake these debates.

2. Structural rules, logics and logical practice. Classical logic, while having been so radically transformed as to warrant the distinction between formal (philosophical) and symbolic (mathematical) logic,⁸¹ maintains itself within the tradition of Aristotelian logic. It can be presented in terms of different axiom systems and rules and is expressible in various calculi, each of which is fully translatable into any other. In some such calculi (Gentzen sequential calculi) axioms are replaced by rules for the

⁷⁹ K. Gödel, ‘Russell’s Mathematical Logic’ in *The Philosophy of Bertrand Russell*, in ed. P.A. Schilpp (Library of Living Philosophers 5) La Salle, Illinois, Open Court Publishing, 1949 at 126.

⁸⁰ Although somewhat technical, and although historians of mathematical logic are now writing more finely tuned accounts of these debates, I think S.C. Kleene’s classic text-book *Introduction to Metamathematics*, Groningen/Amsterdam, Wolters-Noordhoff Publishing/North-Holland Publishing Company, 1972, Ch.III a readable introduction.

⁸¹ While Leibniz is an ancestor figure, the transformation began in the mid-nineteenth century with the work of Boole and the algebraists, gaining impetus and a somewhat different direction with Frege’s *Begriffsschrift* (1879). The two streams merged in the 1930’s. See Jean van Heijenoort ‘Logic as Calculus and Logic as Language’, above n.25.

introduction of logical constants ('operational rules') and rules of a different type, commonly termed 'structural rules'.⁸² The types of rules are clearly distinguishable. The structural rules contain no logical constants ('and', 'or', 'if . . . then' etc.) and can be seen as governing the handling of assumptions only, in one of Gentzen's calculi.

Commonly, just four structural rules are listed. They embody properties of the deducibility relation in classical logic which are built into it prior to what may otherwise be regarded as its 'content', the logical constants. Non-classical logics may well include some of the structural rules, but a distinguishing feature of classical and non-classical logics is that the latter give up one or more of the structural rules whereas classical logic is a singular edifice. There is one and only one classical logic (although it is differently presentable and can be expressed in a variety of calculi) and it employs all of the structural rules. The field of mathematical or symbolic logic is thus divided between classical and non-classical (intuitionistic, many valued, quantum, dialethic, dialectical, relevant, affine and others) logics. There are, certainly, polemical exchanges across that border. Apart from anything else, what is and is not 'logic' might be made an issue here.⁸³ But practice in the field sees logicians (and applied logicians such as theoretical computer scientists) employing whichever logic suits their purpose and regarding that as doing 'usual logic'. Here, as elsewhere, pluralism may be advocated as a favoured ideology. If so it looks to me as if the 'logic' debate is conducted on the familiar terrain of debate between 'conservatives' and 'liberals'.

3. Contraction: One of the structural rules that is of particular interest in the construction of a dialectical and speculative mathematical logic, has recently become the object of quite some activity in theoretical computer science. This rule (with the usual variability of terminology) is called

⁸² Gerhard Gentzen, 'Untersuchungen über das logische Schliessen', *Mathematische Zeitschrift* 41 (1934), 176–210 and 405–231. Translated by M.E. Szabo in *The Collected Papers of Gerhard Gentzen*, Amsterdam and London, North-Holland Publishing Company, 1969.

⁸³ Quine for example, with reference to Gödel's proof of the completeness of first-order predicate logic says: "This calculus [logic] is the basic department of modern formal logic; there are some who even equate it to logic, in a defensible narrow sense of the word". W.V. Quine, 'Kurt Gödel' in *Theories and Things*, Cambridge, Ma., Harvard University Press, 1981, 143–147 at 143. Terminology here is confusing since 'calculus' can be used to mean both a system of logic and its symbolic expression.

‘contraction’. What it allows can be seen as dropping an assumption in a proof where that assumption has been used more than once. Standing behind this permission is an assumption of the re-usability of assumptions, or in other words, having an assumption once is as good as having it twice or more. Since ‘resource consciousness’ or ‘good accounting’ are desirable in constructing systems of logic suitable for computing (given practical limitations of time and memory) logical systems which drop contraction have been devised.⁸⁴

As I understand contraction, in the reasoning that classical logic formalises, the possibility that assumptions may be used up in the ‘act’ of inference is excluded. Perhaps here something of the Hegelian spirit glimmers: a dynamic spirit for which, following an inference (transition), things are not just as before.

4. Bivalence and its Restriction: Perhaps this treatment of assumptions, in being consistent with ideas of unchangeable, eternal truths, supports these ideas and the ideal of truth abstracted from them. Perhaps it goes the other way, i.e. perhaps the rule is a formalisation of that ideal. Perhaps it goes both ways. I want to move on from here to the specific meta-logical assumption of classical logic itself, namely the assumption of bivalence, or truth-definiteness, or more fully of the validity of either-or reasoning *as applied to the truth values*, true and false. As Quine puts it:

Bivalence is a basic trait of our classical theories of nature. It has us positing a true-false dichotomy across all the statements that we can express in our theoretical vocabulary, irrespective of our knowing how to decide them. In keeping with our theories of nature we have viewed all such sentences as having factual content, however remote from observation. In this way simplicity of theory has been served.⁸⁵

Quine’s purpose, in this article, is to point out the ‘price’ of bivalence. “We stalwarts of two-valued logic” he writes,

⁸⁴ This is a technical area to which I know no introduction that is readable without specialist knowledge. The phrase “resource conscious” is taken from A.S. Troelstra, *Lectures on Linear Logic*, Stanford, Center for the Study of Language and Information, 1992 at 1.

⁸⁵ W.V. Quine, ‘What Price Bivalence’ *The Journal of Philosophy* LXXVII (1981), 90–95 at 94.

buy its sweet simplicity at no small price in respect of harboring of undecideables.⁸⁶

The price, apparently, gives value for money: deductive power. Classical logic remains hegemonic. Its main rival, intuitionistic logic, though announced as a revolution,⁸⁷ turned out to be a rather moderate (and useful) reform.⁸⁸ But my point here is that this meta-logical assumption can be and is restricted in many non-classical logics. That is to say its universality is reigned in from all to some sentences. Or if logical bivalence is conceived as inherently universalist, then one would say it is given up, abandoned. It does not then follow that there are no sentences which are not either true or false in logics which restrict bivalence. Bivalence may be provable for *some* sentences by the logic in question. With such an abandonment, *accompanied by* an abandonment of the contraction rule, one gets a ‘contraction-free’ logic within which a certain form of contradiction (e.g. ‘If A then not-A and if not-A then A’) does not ‘trivialise’ the system (as does e.g. ‘A and not-A’), that is, allow anything and everything to be proved by it. Again, the implementation of these acts of abandonment in a system of non-classical logic, takes shape in the presentation of the system. To William Rasch’s playful question “To what is the law of excluded middle subject?”⁸⁹ my non-playful answer is: scrutiny and the possibility of being dropped!

5. The abstraction axiom, antinomies and incompleteness: Deductive power has been met. A formal logical system is also characterised by more or less expressive power. An abstraction axiom is basically a formal

⁸⁶ Ibid at 91.

⁸⁷ By Hermann Weyl: see Davis, above n.48 at 96; see also Constance Reid, *Hilbert*, Berlin, Springer-Verlag, 1970, Ch. XVIII.

⁸⁸ The translation of a meta-logical assumption into theorems or rules of inference of a formal logical system is a tricky business. Intuitionistic logic ‘gives up’ excluded middle (or double negation which is equivalent to excluded middle within intuitionistic logic), and it may be said that, in a certain sense, contraction is restricted, because only one well formed formula is allowed in the succedent of a Gentzen sequent. But a well formed formula according to a schema ‘not-(A and not-A)’ (excluded contradiction) is deducible in intuitionistic logic. Some ingenious fiddling about with axiom systems and it turns out to be no less deductively powerful than classical logic. The moral here is that it does not give a lot of purchase on formal systems to talk about them in terms of the names traditionally given to principles of classical logic.

⁸⁹ William Rasch, *Sovereignty and its Discontents: On the Primacy of Conflict and the Structure of the Political*, London, Birkbeck Law Press, 2004 at 89.

axiom of concept formation and gives expressive power to systems including such an axiom. In natural languages with a subject–predicate structure, the construction of an object from a predicate, as in, for example, a move from ‘is red’ to ‘redness’ is illustrative of the operation permitted (‘objectification’). It enables predication of an objectified predicate, such as ‘redness’. That is, something further can be predicated of ‘redness’ whereas ‘is red’ cannot be used as a subject. Hence the usefulness of the rule. With an unrestricted abstraction axiom one has unlimited expressive power. Historically, its employment goes back to Frege; to his logicist project of reducing arithmetic to logic and his construction of the ideal calculus for that purpose, that is, (roughly! the technical details of Frege’s system were otherwise given) classical predicate logic with abstraction axioms. The drawback is, as the failure of Frege’s project showed, that in some cases, of which Russell’s class is the classic example (the class of classes that are not members of themselves), when used in conjunction with classical logic, an abstraction axiom leads directly into full blown logical antinomy.

In classical logic, the abstraction rule is therefore severely restricted, although it is not given up.⁹⁰ But in a contraction-free logic, there is no passage from a contradiction appearing in the form ‘If A then not-A and if not-A then A’ to the form of contradiction ‘A and not-A’. There is thus no need to restrict abstraction and such logics are said to allow unrestricted concept formation or unrestricted abstraction. When dealing with an object like Russell’s class in such a logic, one will end up not with logical antinomy but with an ‘undecidability’ result.

An undecidability result: *the* undecidability result that passes so many lips, Gödel’s incompleteness theorem, is derived in classical logic for consistent, formal theories (or axiom systems) containing some arithmetic. The first incompleteness theorem establishes that there is a sentence in the language of the theory in question, which is neither provable nor refutable in that theory. It is (roughly) a sentence that says of itself that it is unprovable. The second theorem shows that the consistency of the system cannot be proved within the system. What such results ‘mean’

⁹⁰ The development of type theories as a means of avoiding such antinomies without entirely abandoning abstraction, is part of the history of mathematical logic. Type theories remain a feature of contemporary classical logic as one means of specifying restrictions on abstraction. There are also other, less restrictive, means.

philosophically is contested. It has been taken as an “excellent and beautiful example” of Hegelian dialectic⁹¹ and it has been taken as a decisive refutation of Hegel.⁹² I take it as an event of which the surprise is that a certain proposition is provable within the system for each and every natural number but is neither provable nor disprovable for the totality of all natural numbers. I fear that those who believe or even know that banning ‘totalities’ or ‘closed totalities’ is the way to deal with the threat of totalitarian political systems, will see my surprise as very naive. It is a risk I take.

⁹¹ J.N. Findlay, above p.78; Findlay’s interpretation is backed by the truly remarkable accomplishment of a natural language version of the proof of the first theorem that appeared just twelve years after publication of Gödel’s paper (J.N. Findlay, ‘Goedelian sentences: a non-numerical approach’ *Mind* LI (1942), 259–265).

⁹² J.M. Bochenski, ‘The general sense and character of modern logic’ in Agazzi, above n.52, 3–14 at 14.