

# John von Neumann and the Evolutionary Growth of Complexity

“Vague, unscientific, and imperfect . . .”

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# Burks' Problem: Machine Self-Reproduction

Various Frameworks considered:

- Kinematic Framework
  - Tinker toys (Lego Technic)?
  - Not tractable (at least, not in 1940s/1950s; today maybe simulate in an off the shelf **physics engine**; see also this **16-bit ALU in minecraft!**)
- Cellular ("Tesselation") Framework (Ulam/von Neumann)

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# (Non-)Trivial Self Reproduction? Burks' Criterion

Burks (1970):

*... This result is obviously substantial, but to express its real force we must formulate it in such a way that it cannot be trivialized ...*

*... Consider, for example, a two-state cellular system whose transition function takes a cell into state one when any of its neighbors is in state one. Define an automaton to be any area, even a single cell. A cell in state one then reproduces itself trivially in its neighboring cells ...*

*... Clearly what is needed is a requirement that the self-reproducing automaton have some minimal complexity ...*

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# But ... Herman's Counter Example

Herman (1973):

- 2D Cellular Framework
- Cells combine trivial, crystalline, reproduction with universal turing machine head
- Cells still simpler than in von Neumann model!

*...What the result does show is that the existence of a self-reproducing universal computer-constructor in itself is not relevant to the problem of biological and machine self-reproduction...*

*... Hence, there is a need for new mathematical conditions to insure non-trivial self-reproduction.*

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Langton (1984):

- Critique: Computational criterion too strong rather than too weak?
- Alternative: Non-trivial self reproduction characterised by separate processes of copying and decoding of a machine description.
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*... One of the difficulties in defining what one means by self-reproduction is that certain organizations, such as growing crystals, are self-reproductive by any naive definition of self-reproduction, yet nobody is willing to award them the distinction of being self-reproductive. . .*

*... A way around this difficulty is to say that self-reproduction includes the ability to undergo inheritable mutations as well as the ability to make another organism like the original. . .*

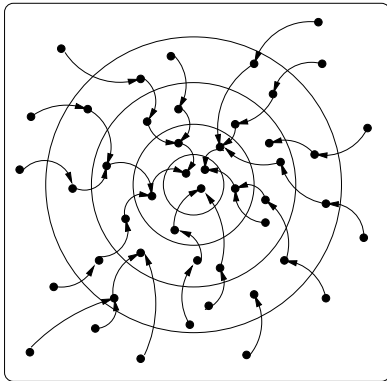
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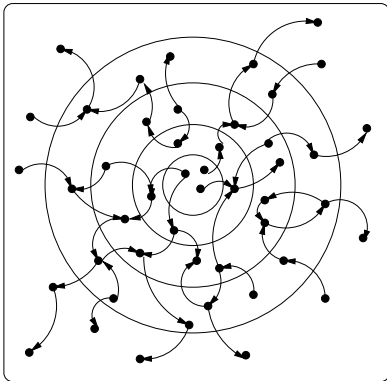
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# Degeneration of Complexity (Engineering)





# Growth of Complexity (Biology)



## von Neumann's (*real*) Problem ...

- How can machines manage to construct other machines more “complex” than themselves, in a general and open-ended way — i.e., with the potential for unbounded evolutionary growth of complexity.
- What might count as a “solution”?
  - Exhibit a class of constructing machines, spanning a wide range of complexities, such that the whole class is connected by the relative construction (mutation) network.
  - For good measure, require that every machine in the class also be SR (every machine has a self-construction loop). This is necessary (not sufficient) for Darwinian selection.

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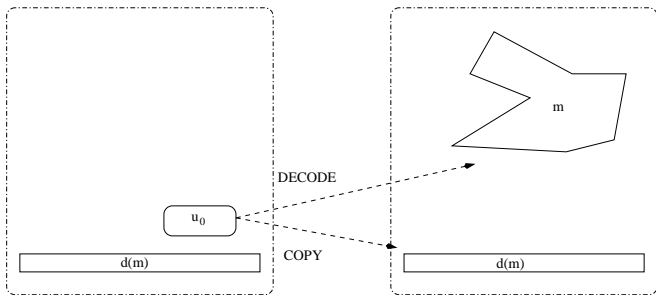
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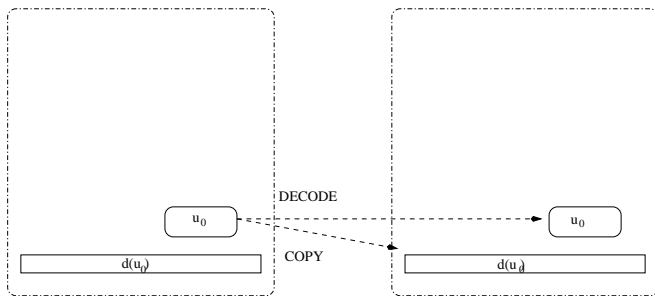
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# The General Constructive Automaton



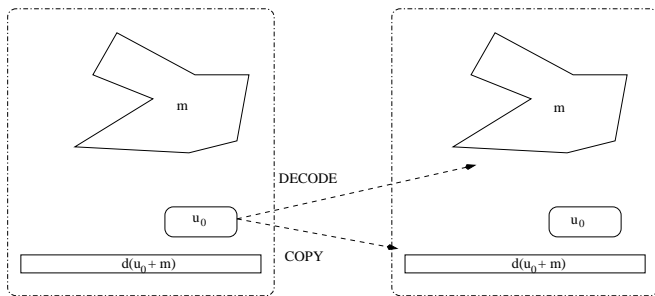
$$(u_0 \oplus d(m)) \rightsquigarrow (m \oplus d(m))$$

# Von Neumann SR: Minimal Case



$$(u_0 \oplus d(u_0)) \rightsquigarrow (u_0 \oplus d(u_0))$$

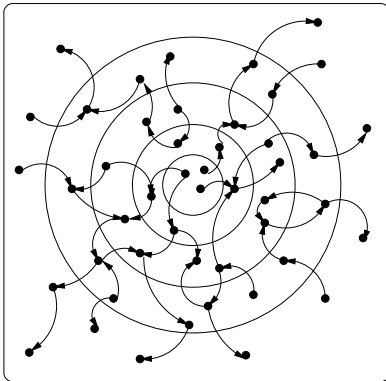
# Von Neumann SR: Generic Case



$$((u_0 \oplus m) \oplus d(u_0 \oplus m)) \rightsquigarrow ((u_0 \oplus m) \oplus d(u_0 \oplus m))$$



# Growth of Complexity (von Neumann)



# Conclusions: Looking Backward

- Is this the *first* solution?
- Is it the *only* solution?
- Is it the *simplest* solution?

# Conclusions: Looking Forward

- Complexity?
- Individuality?
- Origins?
- Evolutionary growth of complexity?

# References

- Burks, A. W. (1970). Von neumann's self-reproducing automata. In Burks, A. W., editor, *Essays on Cellular Automata*, pages 3–64 (Essay One). University of Illinois Press, Urbana.
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## Related Online Resources

- Source files for presentation ( $\text{\LaTeX}$ +beamer):
  - <http://alife.rince.ie/talks/altona-2011/altona-2011-src.zip>
- Full Paper (Artificial Life, Vol. 6, Issue 4, Fall 2000, pp. 347–361):
  - <http://alife.rince.ie/bmcm-2000-01/>
- Rince/DCU Alife Laboratory:
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